Handout (Limits and Continuity)

(Schwartz; Math 157)

Work the problems on your own paper just like a normal homework assignment.

1. Sketch a possible graph of $f(x)$ given that $\lim_{x\to 1^+} f(x) = 2$, $\lim_{x\to 1^-} f(x) = 5$, and $f(1) = 0$.

2. Sketch a graph of $g(x) = \begin{cases} |x|; x \le 2 \\ \sqrt{x}; x > 2 \end{cases}$ and use it find each of the following limits. Then check your answers by finding the limits numerically.
   (a) $\lim_{x\to 2^-}(g(x))$
   (b) $\lim_{x\to 2^+}(g(x))$
   (c) $\lim_{x\to 2}(g(x))$
   (d) $\lim_{x\to -2}(g(x))$

3. For each of the following, find the left- and right-hand limits at the given value of *x* then determine whether the function is continuous at that value of *x*. For those that are not continuous, state what the problem is in terms of limits.
   (a) $f(x) = \frac{x+2}{|x+2|}$ at $x = 2$.
   (b) $f(x) = \frac{x+2}{|x+2|}$ at $x = -2$.
   (c) $g(x) = \begin{cases} x^2; x \le 3 \\ x + 9; x > 3 \end{cases}$ at $x = 3$
   (d) $h(x) = \begin{cases} \frac{x}{x^2+5x}; x \ne 0 \\ 0; x = 0 \end{cases}$ at $x = 0$

4. You should have found that $g(x) = \begin{cases} x^2; x \le 3 \\ x + 9; x > 3 \end{cases}$ is not continuous at $x = 3$. Pick a value for *C* so that $g(x) = \begin{cases} x^2; x \le 3 \\ x + C; x > 3 \end{cases}$ is continuous at $x = 3$.

5. You should have found that $h(x) = \begin{cases} \frac{x}{x^2+5x}; x \ne 0 \\ 0; x = 0 \end{cases}$ is not continuous at $x = 0$. However, you could make it continuous if you define $h(0)$ to be something other than zero. What would $h(0)$ have to be in order to make $h(x)$ continuous?

6. Find values for *m* and *n* in the function $f(x) = \begin{cases} 2x - m; x \le 3 \\ -x + n; x > 3 \end{cases}$ so that $\lim_{x\to 3} f(x) = 5$. Then sketch the graph of $f(x)$.